AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1. (Currently amended) A method for finding zeros of a function, f, within an interval, X, using the interval version of Newton's method, wherein f' is the derivative of the function f, the method comprising:

receiving a representation of the interval X, the representation including a first floating-point number, a, representing the left endpoint of X, and a second floating-point number, b, representing the right endpoint of X;

storing the representation in a memory of the computer system;

performing an interval Newton step on X, wherein the point of expansion is the midpoint, x, of the interval X, and wherein performing the interval Newton step involves evaluating $f(x)$ to produce an interval result $f^I(x)$; and

if $f^I(x)$ contains zero,

evaluating $f(a)$ to produce an interval result $f^I(a)$,

evaluating $f(b)$ to produce an interval result $f^I(b)$,

evaluating a termination condition for the processing of the current interval X, wherein the termination condition is TRUE if a number of conditions are satisfied, including if $f^I(a)$ contains zero and if $f^I(b)$ contains zero, and

if the termination condition is TRUE, terminating the processing of the current interval X, and recording X in the memory as a final bound.

2. (Original) The method of claim 1, wherein if $f^I(a)$ does not contain zero, evaluating $f(a)$ additionally involves performing an interval Newton step wherein the point of expansion is a.

3. (Original) The method of claim 1, wherein if $f^I(b)$ does not contain zero, evaluating $f(b)$ additionally involves performing an interval Newton step wherein the point of expansion is b.

4. (Original) The method of claim 1, wherein if $f^I(x)$ contains zero and $f'(X)$ contains zero, the termination condition for processing the current interval X is TRUE if $f^I(a)$ contains zero, $f^I(b)$ contains zero, $f^I(x_1)$ contains zero and $f^I(x_2)$ contains zero;

wherein x_1 is the midpoint between a and x; and

wherein x_2 is the midpoint between x and b.

5. (Original) The method of claim 4, wherein if $f^I(x)$ contains zero and if $f'(X)$ contains zero, and if either $f^I(x_1)$ or $f^I(x_2)$ does not contain zero, the method further comprises:

splitting the interval X in half; and

applying the interval Newton method to each half separately.

6. (Original) The method of claim 1, wherein if $f^I(x)$ does not contain zero and if $f'(X)$ contains zero, the method further comprises terminating the processing of the current interval X, and recording X as a final bound on condition that:

the width of the interval X divided by the magnitude of the interval X is less than a first threshold value; and

the magnitude of $f(X)$ is less than a second threshold value.

7. (Original) The method of claim 1, wherein if a given Newton step does not reduce the width of an interval by at least half, the method further comprises splitting the interval in half and applying the interval Newton method to each of the two halves separately.

8. (Original) The method if claim 1, wherein if an interval Newton step results in two intervals, the method further comprises applying the interval Newton method to each of the two intervals separately.

9. (Original) The method of claim 1, wherein if the result of an interval Newton step is the empty interval, the method returns to process another interval.

10. (Original) The method of claim 1, wherein if $f'(a)$ contains zero and if $f'(b)$ contains zero, the method returns to process another interval.

11. (Original) The method of claim 1,

wherein if $f'(a)$ does not contain zero or if $f'(b)$ does not contain zero, the method further comprises performing an interval Newton step wherein the point of expansion is the midpoint of the interval X; and

wherein if the result of the interval Newton step about the midpoint is the empty interval, the method returns to process another interval.

12. (Original) The method of claim 11, further comprising terminating the processing of the current interval X after a predetermined number of iterations.

13. (Original) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for finding zeros of a function, f, within an interval, X, using the interval version of

Newton's method, wherein f' is the derivative of the function f, the method comprising:

receiving a representation of the interval X, the representation including a first floating-point number, a, representing the left endpoint of X, and a second floating-point number, b, representing the right endpoint of X;

performing an interval Newton step on X, wherein the point of expansion is the midpoint, x, of the interval X, and wherein performing the interval Newton step involves evaluating $f(x)$ to produce an interval result $f^I(x)$; and

if $f^I(x)$ contains zero,

evaluating $f(a)$ to produce an interval result $f^I(a)$,

evaluating $f(b)$ to produce an interval result $f^I(b)$,

evaluating a termination condition for the processing of the current interval X, wherein the termination condition is TRUE if a number of conditions are satisfied, including if $f^I(a)$ contains zero and if $f^I(b)$ contains zero, and

if the termination condition is TRUE, terminating the processing of the current interval X, and recording X as a final bound.

14. (Original) The computer-readable storage medium of claim 13, wherein if $f^I(a)$ does not contain zero evaluating $f(a)$ additionally involves performing an interval Newton step wherein the point of expansion is a.

15. (Original) The computer-readable storage medium of claim 13, wherein if $f^I(b)$ does not contain zero, evaluating $f(b)$ additionally involves performing an interval Newton step wherein the point of expansion is b.

16. (Original) The computer-readable storage medium of claim 13, wherein if $f^I(x)$ contains zero and $f'(X)$ contains zero, the termination condition for processing the current interval X is TRUE if $f^I(a)$ contains zero, $f^I(b)$ contains zero, $f^I(x_1)$ contains zero and $f^I(x_2)$ contains zero;

wherein x_1 is the midpoint between a and x; and

wherein x_2 is the midpoint between x and b.

17. (Original) The computer-readable storage medium of claim 16, wherein if $f^I(x)$ contains zero and if $f'(X)$ contains zero, and if either $f^I(x_1)$ or $f^I(x_2)$ does not contain zero, the method further comprises:

splitting the interval X in half; and

applying the interval Newton method to each half separately.

18. (Original) The computer-readable storage medium of claim 13, wherein if $f^I(x)$ does not contain zero and if $f'(X)$ contains zero, the method further comprises terminating the processing of the current interval X, and recording X as a final bound on condition that:

the width of the interval X divided by the magnitude of the interval X is less than a first threshold value; and

the magnitude of $f(X)$ is less than a second threshold value.

19. (Original) The computer-readable storage medium of claim 13, wherein if a given Newton step does not reduce the width of an interval by at least half, the method further comprises splitting the interval in half and applying the interval Newton method to each of the two halves separately.

20. (Original) The computer-readable storage medium if claim 13, wherein if an interval Newton step results in two intervals, the method further comprises applying the interval Newton method to each of the two intervals separately.

21. (Original) The computer-readable storage medium of claim 13, wherein if the result of an interval Newton step is the empty interval, the method returns to process another interval.

22. (Original) The computer-readable storage medium of claim 13, wherein if $f^I(a)$ contains zero and if $f^I(b)$ contains zero, the method returns to process another interval.

23. (Original) The computer-readable storage medium of claim 13,

wherein if $f^I(a)$ does not contain zero or if $f^I(b)$ does not contain zero, the method further comprises performing an interval Newton step wherein the point of expansion is the midpoint of the interval X; and

wherein if the result of the interval Newton step about the midpoint is the empty interval, the method returns to process another interval.

24. (Original) The computer-readable storage medium of claim 13, wherein the method further comprises terminating the processing of the current interval X after a predetermined number of iterations.

25. (Currently amended) An apparatus that finds zeros of a function, f, within an interval, X, using the interval version of Newton's method, wherein f' is the derivative of the function f, the apparatus comprising:

a receiving mechanism that is configured to receive a representation of the interval X, the representation including a first floating-point number, a,

representing the left endpoint of X, and a second floating-point number, b, representing the right endpoint of X;

a storing mechanism configured to store the representation in a memory of the computer system;

an interval Newton mechanism that is configured to perform an interval Newton step on X, wherein the point of expansion is the midpoint, x, of the interval X, and wherein performing the interval Newton step involves evaluating $f(x)$ to produce an interval result $f^I(x)$; and

wherein if $f^I(x)$ contains zero, the interval Newton mechanism is configured to,

evaluate $f(a)$ to produce an interval result $f^I(a)$,

evaluate $f(b)$ to produce an interval result $f^I(b)$,

evaluate a termination condition for the processing of the current interval X, wherein the termination condition is TRUE if a number of conditions are satisfied, including if $f^I(a)$ contains zero and if $f^I(b)$ contains zero, and

if the termination condition is TRUE, to terminate the processing of the current interval X, and to record X in the memory as a final bound.

26. (Original) The apparatus of claim 25, wherein while evaluating $f(a)$ to produce the interval result $f^I(a)$, the interval Newton mechanism is additionally configured to perform an interval Newton step wherein the point of expansion is a if $f^I(a)$ does not contain zero.

27. (Original) The apparatus of claim 25, wherein while evaluating $f(b)$ to produce the interval result $f^I(b)$ the interval Newton mechanism is additionally

configured to perform an interval Newton step wherein the point of expansion is b if $f^I(b)$ does not contain zero.

28. (Original) The apparatus of claim 25, wherein if $f^I(x)$ contains zero and $f'(X)$ contains zero, the termination condition for processing the current interval X is TRUE if $f^I(a)$ contains zero, $f^I(b)$ contains zero, $f^I(x_1)$ contains zero and $f^I(x_2)$ contains zero;

wherein x_1 is the midpoint between a and x; and

wherein x_2 is the midpoint between x and b.

29. (Original) The apparatus of claim 28, wherein if $f^I(x)$ contains zero and if $f'(X)$ contains zero, and if either $f^I(x_1)$ or $f^I(x_2)$ does not contain zero, the interval Newton mechanism is additionally configured to:

split the interval X in half; and to

apply the interval Newton method to each half separately.

30. (Original) The apparatus of claim 25, wherein if $f^I(x)$ does not contain zero and if $f'(X)$ contains zero, the interval Newton mechanism is additionally configured to terminate the processing of the current interval X, and to record X as a final bound on condition that:

the width of the interval X divided by the magnitude of the interval X is less than a first threshold value; and

the magnitude of $f(X)$ is less than a second threshold value.

31. (Original) The apparatus of claim 25, wherein if a given Newton step does not reduce the width of an interval by at least half, the interval Newton mechanism is additionally configured to split the interval in half, and to apply the interval Newton method to each of the two halves separately.

32. (Original) The apparatus if claim 25, wherein if an interval Newton step results in two intervals, the interval Newton mechanism is additionally configured to apply the interval Newton method to each of the two intervals separately.

33. (Original) The apparatus of claim 25, wherein if the result of an interval Newton step is the empty interval, the interval Newton mechanism is additionally configured to return to process another interval.

34. (Original) The apparatus of claim 25, wherein if $f^I(a)$ contains zero and if $f^I(b)$ contains zero, the interval Newton mechanism is additionally configured to return to process another interval.

35. (Original) The apparatus of claim 25,

wherein if $f^I(a)$ does not contain zero or if $f^I(b)$ does not contain zero, the interval Newton mechanism is additionally configured to perform an interval Newton step wherein the point of expansion is the midpoint of the interval X; and

wherein if the result of the interval Newton step about the midpoint is the empty interval, the interval Newton mechanism is additionally configured to return to process another interval.

36. (Original) The apparatus of claim 35, wherein the interval Newton mechanism is additionally configured to terminate the processing of the current interval X after a predetermined number of iterations.